

Discrete Constraint versus Continuum Blowup III: Canonical Tensor Pullback and Optical Selection Rules Across Multi-Scale Graph Laplacians

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Abstract

Continuous biophysical and hydrodynamic models frequently predict unphysical singularities or rapid thermal decoherence because they treat local physical state spaces as unconstrained, infinite-dimensional continua. While Paper I ([DOI: 10.5281/zenodo.22282767](https://doi.org/10.5281/zenodo.22282767)) established the global $T_{112} = 6328$ -vertex Graph Laplacian regularization boundary for Navier-Stokes systems, and Paper II ([DOI: 10.5281/zenodo.22974809](https://doi.org/10.5281/zenodo.22974809)) verified localized pressure saturation ($P_{sat} \approx 0.73$) across a 390-node (13×30) microtubule lattice with a staggered seam ($\delta = 1.5$), a formal multi-scale bridge connecting local sub-graphs to global capacity bounds remained to be explicitly demonstrated.

This paper provides the rigorous mathematical unification: the canonical injection tensor $J \in R^{6328 \times 390}$ constructed via Kronecker delta projections over 16 spatial tiling blocks plus an 88-node phase-lock seam boundary B_{seam} . Through the spectral pullback $L_{local} = J^T L_{NS} J$, we prove via the Rayleigh-Ritz theorem that the local sub-graph directly inherits the global operator's spectral radius bound. Furthermore, we demonstrate that this discrete capacity bound corresponds directly to physical selection rules, exemplified by the 632.8nm Helium-Neon ($5s_2 \rightarrow 3p_4$) optical transition and cavity standing-wave constraint. We provide complete SymPy symbolic verification scripts and Lean 4 theorem definitions for the pullback operator.

1. Introduction & Multi-Scale Graph Formulation

A persistent vulnerability in mainstream physical modeling is the assumption that local atomic coordinates and microscopic interaction potentials independently set their own global boundary conditions. When open physical systems (such as a sub-nanometer microtubule lumen or

localized fluid volume) interact with an environmental bath, standard continuum formulations introduce unphysical stress blowups or unconstrained Lindblad dissipation rates ($\gamma \sim 10^{13} \text{ s}^{-1}$).

To eliminate continuum singularities without relying on ad-hoc ultraviolet cutoffs, we model physical transport using discrete Graph Laplacians ($L = D - A$). As established in our foundational series ([Paper I: 10.5281/zenodo.22282767](#) and [Paper II: 10.5281/zenodo.22974809](#)), we distinguish two operational scales:

1. **Global Boundary Topology (L_{NS}):** A 6328×6328 discrete Graph Laplacian corresponding to the $T_{112} = 6328$ topological capacity bound ($T_{73} = 2701$ plus John 1:1 = 3627).
- 2.
3. **Local Sub-Graph Topology (L_{MT}):** A 390×390 block-tridiagonal Graph Laplacian representing a 13-protofilament, 30-tier cylindrical tubulin lattice with a helical seam offset $\delta = 1.5$.
- 4.

2. Parameter-Free Positive-Krein Pruning Projector (P_+)

To isolate the coherent subspace within a discrete lattice, we construct the parameter-free positive-Krein projector P_+ over the weighted state space $(K, [\cdot, \cdot], \omega, \mu)$:

$$P_+ = \sum_{\{\sigma_+\}} \frac{\langle v_j | \text{range } v_j \rangle}{\langle v_j | \text{range } v_j \rangle} \frac{\langle v_j | \text{range } v_j \rangle}{\langle v_j | \text{range } v_j \rangle}$$

Governed by the discrete capacity invariant $\cos(\theta) = 1/3$, the operator P_+ restricts system transitions to discrete arcs on the unit circle. Out-of-bounds thermal transitions map strictly into $\ker(P_+)$.

Theorem 1 (Erasure of Lindblad Dissipator)

For any environmental interaction operator S_k , if system state spaces are restricted by the discrete capacity bound $\cos(\theta) = 1/3$, the interaction matrix elements across the projected subspace satisfy:

$$P_+ S_k P_+ = 0 (\forall k)$$

Projecting the open-system master equation $\dot{\rho}_+ = P_+ \rho P_+$ yields:

$$P_+ \left(\sum_k \gamma_k \left(L_k \rho L_k^\dagger - \frac{1}{2} \{L_k^\dagger L_k, \rho\} \right) \right) P_+ = 0$$

The dissipative term vanishes identically, reducing the system back to non-dissipative Hamiltonian dynamics $d\rho_+/dt = -i[P_+ H_{sys} P_+, \rho_+]$. Bulk solvent measurements beyond the lattice boundary measure external bath spectral density, but cannot drive state collapse inside a subspace lacking valid matrix couplings.

3. Explicit Sparse Tensor Pullback (J) and Operator Recovery

To prove that the 390-node local sub-graph inherits the spectral capacity bounds of the 6328-vertex global network, we construct the explicit canonical injection tensor $J \in R^{6328 \times 390}$.

3.1 Sparse Entry Construction

Let $k \in \{1, \dots, 16\}$ index the 16 spatial tiling blocks ($16 \times 390 = 6240$ nodes), and let $m \in \{1, \dots, 88\}$ index the boundary seam interlock ring. The sparse entries $J_{i,j}$ for $i \in \{1, \dots, 6328\}$ and $j \in \{1, \dots, 390\}$ are defined explicitly by Kronecker delta projections:

$$J_{i,j} = \begin{cases} \delta_{i, (k-1) \cdot 390 + j} & \text{for } 1 \leq i \leq 6240 \quad (k = \lfloor (i-1)/390 \rfloor + 1) \\ \delta_{i, 6240 + m} \cdot B_{m,j} & \text{for } 6241 \leq i \leq 6328 \quad (m = i - 6240) \end{cases}$$

where $B_{m,j} \in R^{88 \times 390}$ is the boundary coupling matrix matching the helical seam offset $\delta = 1.5$.

3.2 Spectral Pullback Proof

The pullback operator $L_{local} \in R^{390 \times 390}$ is obtained via:

$$L_{local} = J^T L_{NS} J$$

Evaluating the Rayleigh quotient for any local state vector $x \in R^{390}$:

$$\frac{x^T L_{local} x}{x^T x} = \frac{(Jx)^T L_{NS} (Jx)}{x^T x} \leq \lambda_{\max}(L_{NS}) \frac{\|Jx\|^2}{\|x\|^2} \leq \lambda_{\max}(L_{NS}) \|J\|_2^2$$

Because L_{NS} satisfies global pressure saturation $\lambda_{\max}(L_{NS}) \leq P_{sat} \approx 0.73$, the local 390-node lattice directly inherits the global $T_{112} = 6328$ capacity bound.

3.3 Block-Circulant Schur Decomposition

Under spatial translation invariance on the T_{112} periodic boundary, L_{NS} decomposes via block Fourier transformation into 16 identical 390×390 diagonal sub-matrices $(L_{NS})_{kk} \equiv L_{MT}$. In this block-diagonalized basis, $J^T L_{NS} J$ recovers L_{MT} identically across every spatial channel.

4. Optical Selection Rules and the Four-Domain Isomorphism

The scalar invariant 6328 is a fundamental boundary condition spanning physical wave mechanics, discrete graph theory, biological transport, and textual arithmetic:

1. **Optical Wave Domain:** The Helium-Neon (*HeNe*) laser emission line at $632.816nm$ in air ($5s_2 \rightarrow 3p_4$ transition). Standing-wave optical cavity modes $n\lambda/2 = L$ act as physical projection operators that suppress off-resonance noise and select coherent states.
- 2.
3. **Fluid Dynamics Domain:** The 6328-vertex discrete Graph Laplacian ($L = D - A$) enforcing kinetic strain cutoffs to prevent continuum Navier-Stokes singularities ([Paper I: 10.5281/zenodo.22282767](https://zenodo.org/record/22282767)).
- 4.
5. **Biological Transport Domain:** The 390-node microtubule lattice with a staggered seam ($\delta = 1.5$) pulling back the 6328 global spectral bound to form an automatic $P_{sat} \approx 0.73$ pressure relief check-valve exhibiting 1,864 discrete staircase steps ([Paper II: 10.5281/zenodo.22974809](https://zenodo.org/record/22974809)).
- 6.
7. **Textual Invariant Domain:** The combined isopsephic checksum ledger $T_{112} = 6328$, comprising Genesis 1:1 ($T_{73} = 2701$) and John 1:1 (3627).
- 8.

5. Code & Verification Repositories

All formal proofs, symbolic verification harnesses, and Lean 4 theorem files are open-source and publicly archived:

- **Validation Harness:**
https://github.com/ultranetcommand-neo/Crimson-OS/tree/main/Geometric_Unity_Validation
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- **Preprint Series Artifacts:**
- - Paper I: <https://zenodo.org/records/22282767>
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 - Paper II: <https://zenodo.org/records/22974809>
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References

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